Finite Element Methods against Limit Equilibrium Approaches for Slope Stability Analysis

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ABSTRACT

A number of powerful numerical models, including limit equilibrium and finite element (FE) methods, have been developed for slope stability analysis in recent decades. The limit equilibrium method contains several limitations, yet is considered the most common approach. However, the advancement of technology has increased the use of the finite element method as it embraces a wider range of features. The limit equilibrium software, SLOPE/W, and the FE program PLAXIS are two common software programs currently employed in geotechnical engineering. Slope analysis using the limit equilibrium method involves a series of slip surfaces dividing ground into vertical slices, and using the static equilibrium equations to calculate the factor of safety (FOS) and stresses for each slice. PLAXIS requires the input of soil properties and elastic-plastic parameters of elements. In this study the properties of a heterogeneous slope, consisting of general fill embankment over soft, slightly over-consolidated clay is implemented in each program. The FOS of slopes is determined for subsequent design requirements, and results are analysed and comparisons are conducted. The effect of Young's modulus on the FOS is also discussed. Recommendations are provided based on the results and previously published findings. The contribution of this paper is beneficial to geotechnical engineers, as it discusses the suitability and limitations of each method and assesses reliability of model outputs for slope stability analyses.

Keywords: finite element method, limit equilibrium, slope stability

1 INTRODUCTION

As the demand for infrastructure and natural resources continues to expand, the requirement for excavation and road construction is intensified. In addition, natural disasters, such as landslides and earthquakes, are costly and important problems faced by geotechnical engineers and geologists. According to Bromhead (1992), obtaining an easy and accurate method to analyse slope stability can have an important impact on the safety assessment of construction sites and natural slopes. The stability of a slope is an important issue for all stakeholders involved before, during and after construction. A small difference in the factor of safety (FOS) can result in a large difference in construction costs if slope stabilisation techniques are to be altered. This is significant, as currently there is not clear evidence to suggest, which method produces the most acceptable results.

Soil slope instability concerning infrastructure is an ongoing problem, as slope failures endanger public safety and lead to costly repair work. A number of powerful design software packages have been developed for slope stability analysis in recent decades. These programs include the Limit Equilibrium (LE) and Finite Element (FE) methods of slope stability analysis. The limit equilibrium method contains several limitations and inconsistencies, yet is considered the most common approach. However, as expressed by Hammouri et al. (2008), the advancement of technology has increased the use of the FE methods as it dominates a wider range of features. As Aryal (2008) stated, due to the progress of technology, FE codes have simplified the slope stability analysis. SLOPE/W and PLAXIS are two common software programs currently employed by geotechnical engineers within Australia and other countries. SLOPE/W and PLAXIS are used for the limit equilibrium and FE methods, respectively. Each program is employed to determine the factor of safety of slopes and their subsequent design requirements. Analysing and comparing the results of each will enable the determination of which program is more accurate, depending on the information required.

This paper compares the FE method with the limit equilibrium method for the slope stability analysis. The effect of elastic modulus (E) on the slope stability is also investigated in the predicted results. The findings of this study can assist the practising engineers in comparing these methods and enabling them to use the most applicable and accurate process in future projects.

2 SLOPE STABILTY METHODS AND MODELLING TOOLS

For many decades, the limit equilibrium method has been the most commonly used approach in solving geotechnical engineering problems; applying the perfectly plastic Mohr-Coulomb criterion (Duncan and Wright, 2005; Chen and Liu 1990; Hammouri et al., 2008). Once the appropriate soil properties and the slope geometry are established, stability calculations are undertaken to ensure the forces causing the slope to fail are significantly less than the resisting forces. These calculations involve computing a FOS under a limit equilibrium analysis procedure, using the equations of static equilibrium. The fundamental assumption is that failure occurs through sliding of a block or mass along a slip surface.

Slope analysis using the limit equilibrium method has been refined using the method of vertical slices. As GEO-SLOPE International (2004) explained, the stability analysis involves passing a slip surface through the soil mass and dividing the inscribed section into vertical slices. For the assumed slip surface, the static equilibrium equations are used to calculate the FOS and stresses for each slice. As Duncan and Wright (2005) explained, the Morgenstern-Price procedure preferred to be adopted in the analysis of the LE method, as it satisfies all the requirements for the static equilibrium.

The FE approach is a feasible method to analyse slopes with inhomogeneous soils, irregular geometry and arbitrary water flow pattern. The FE approach involves transferring the entire mass of the slope into a finite number of elements through the assistance of generated mesh. It has been suggested that the FE method offers greater benefits over the limit equilibrium method. Hammouri et al. (2008) compared the results of various Limit Equilibrium methods with the results from the FE analysis and recommended that the FE method was of more practical use. Although the FE method possesses the available technology to overcome the limit equilibrium method deficiencies, it also introduces new complications in slope stability analysis problems. As expressed by Zheng et.al. (2008) and Griffiths and Lane (1999) some limitations of the FE method for the slope stability analysis include:

- Producing variable FOS results depending on the condition selected.
- The interpretation of results is an issue as users must rely on their intuition and experience to understand the ability of the model to accurately predict the physical slopes behaviour.
- A complex approach with a lack of understanding and uncertainty in results.
- A well trained user with modelling experience is preferred to execute complex analyses.
- Input parameters are not regularly measured and the accessibility of these data is generally poor.

This study is comparing two of the main software programs used in the Geotechnical Engineering industry to analyse the stability of slopes. They are: (1) GEO-SLOPE International Ltd, SLOPE/W Version 7.17 and (2) PLAXIS Professional Version 9. The Morgenstern-Price method is employed in the limit equilibrium method and cohesion-friction angle reduction method is used in the finite element approach as they are the most frequently used, satisfying both force and moment equilibriums and apply to almost all soil profiles and slope geometries. In comparison to PLAXIS software, SLOPE/W does not consider E or Poisson's ratio of the soil.

3 NUMERICAL ANALYSIS

First a series of analysis have been conducted for a simple homogeneous slope. The findings indicated that the difference in the FOS and locations of the critical slip surface are negligible and both FE and limit equilibrium methods are identical for the engineering analysis. The geotechnical model adopted in the analysis of an inhomogeneous slope is illustrated in Figure 1. The depth of the soft clay plus the hard crust is 10m at the considered location. Bedrock is assumed to underlie the soft clay, which is modelled as fixed boundary conditions.

The properties of the embankment fill material have been selected for common cases based on data taken from previously published investigations. The embankment height has been set to 5m where, the unit weight = 21kN/m^3, the Young's modulus = 15MPa, the Poisson's ratio = 0.25, the cohesion =

5kPa and the friction angle = 30°. For undrained analysis the clay sub-layers properties have been assumed as: depth = 10m, the undrained friction angle (ϕ_u) = 0°, the saturated unit weight (γ_s) = 18.7kN/m^3 and the over consolidation ratio (OCR) = 2. The shear strength has been 15kPa for the top 1m and then changing linearly form 3 to 34kPa to the depth of 10m. The adopted design parameters for the drained clay layer have been: the effective unit weight = 8.9kN/m^3, OCR = 2, the effective cohesion = 23 kPa and the friction angle = 20°.

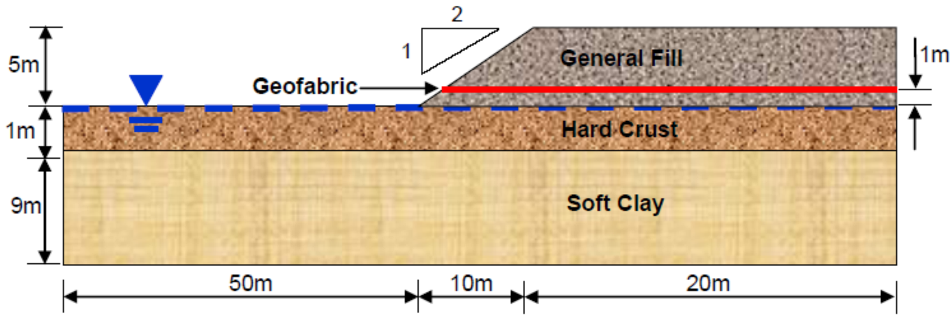


Figure 1. Embankment Profile (not to scale)

PLAXIS requires the input of the Young's modulus (E) and the Poisson's ratio of the soil. As the depth of the clay layer increases, the value of E for clay increases too. A range of E_{ref} values and their increments have been given for the clay layer. The summarised data for the drained and undrained cases are presented in Table 1. The Poisson's ratios for the drained and undrained analyses have been taken up 0.3 and 0.49, respectively.

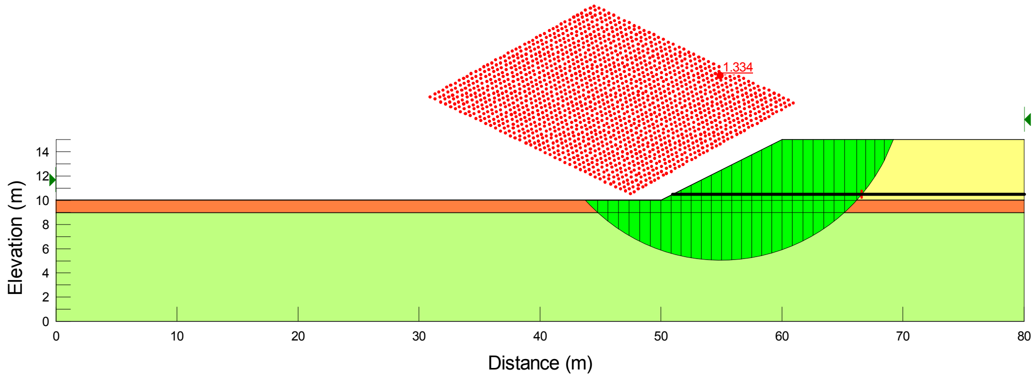
Table 1: Adopted soil properties for drained and undrained cases in PLAXIS

Case	**Soil**	E_{ref} (MPa)	$E_{increment}$ (MPa/m)	c_{ref} (kN/m^2)	E_{ref} (MPa)	$E_{increment}$ (MPa/m)	c_{ref} (kN/m^2)	$c_{increment}$ (kN/m^2/m)
		Drained Clay			**Drained Clay**			
1	Crust	7.5	-	23	8.6	-	15	-
	Clay	1.7	1.7	23	2.0	2.0	3.41	3.41
2	Crust	11.1	-	23	12.8	-	15	-
	Clay	2.5	2.5	23	2.9	2.9	3.41	3.41
3	Crust	14.6	-	23	16.9	-	15	-
	Clay	3.3	3.3	23	3.8	3.8	3.41	3.41
4	Crust	18.2	-	23	21.0	-	15	-
	Clay	4.1	4.1	23	4.8	4.8	3.41	3.41

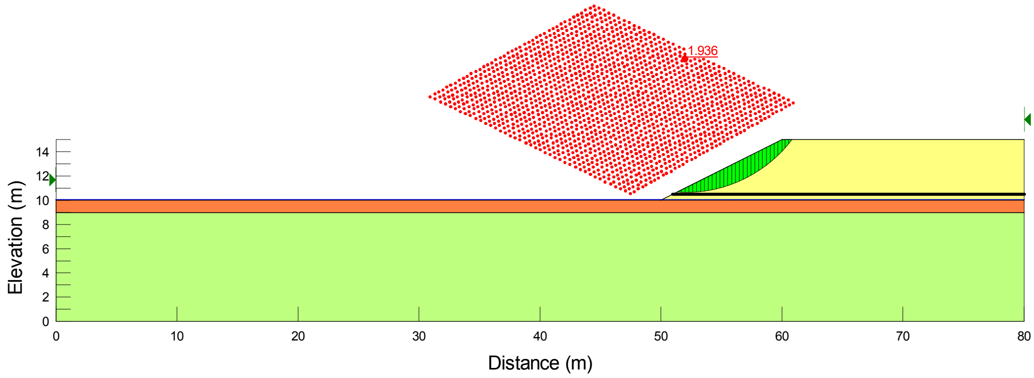
This analysis adopted a layer of geotextiles (geofabrics) to assist in stabilising the embankment. For the SLOPE/W Morgenstern-Price analysis, a FOS equal to 1.3 has been used for the geofabrics. A maximum load of 450kN will therefore produce a working load of 346kN. This factored load is equal to the tensile geofabrics load used in the PLAXIS Phi-c reduction analysis. In PLAXIS, it is assumed that the strength of the geofabrics is 346kN/m with an elastic axial stiffness (EA) of 3460kN/m for 10% strain.

<u>SLOPE/W Analyses</u>: In the limit equilibrium SLOPE/W analysis, the undrained shear strength parameters have been adopted using the SHANSEP technique. As Duncan and Wright (2005) stated, the SHANSEP technique involves "consolidating samples to effective stresses that are higher than the

in-situ stresses and interpreting the measured strengths". Figure 2 illustrates the critical slip surfaces for the SLOPE/W analysis in the undrained and drained cases.



(a)



(b)

Figure 2. SLOPE/W critical slip surface (a) undrained analysis, and (b) drained analysis

<u>PLAXIS Analyses</u>: The mesh used in these analyses is presented in Figure 3. A fine elemental mesh with 15-node triangular elements was employed for the purpose of these analyses. Generally, a smaller element will produce an expensive and more accurate solution. According to Ghiassian et al. (2009) smaller elements are required in areas where stress is concentrated; evident along the geofabrics. The elements can be made larger, further away from these stress concentrated regions without the accuracy being significantly sacrificed .

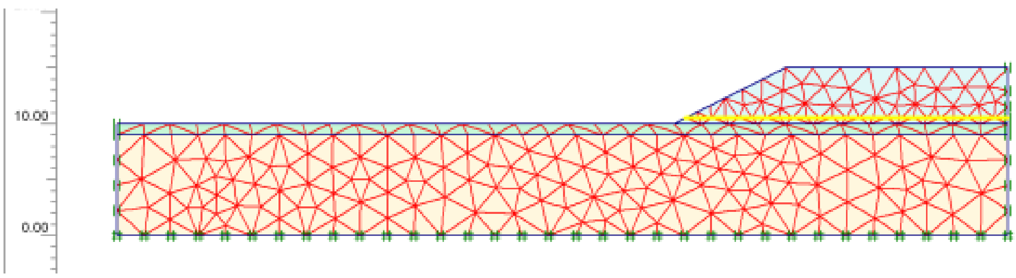


Figure 3. PLAXIS finite element mesh

In the FE Modelling after setting up the initial ground model, two stages have been proposed: embankment construction stage (plastic analysis) and the factor of safety calculation stage (Phi-c reduction analysis). Figure 4 illustrates a typical critical slip surface for a specific value of E value for the PLAXIS analysis in the undrained case.

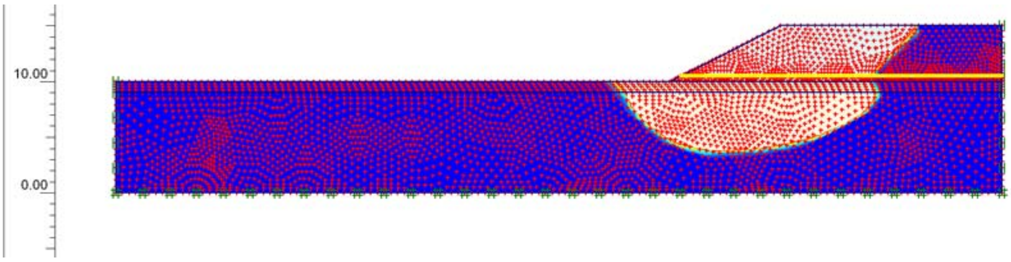


Figure 4. PLAXIS critical slip surface for an undrained analysis (typical)

4 RESULTS AND DISCUSSION

Table 2 outlines the FOS results achieved from the FE analyses for different adopted modulus of elasticity values. The results clearly indicate that E has an insignificant effect on the FOS denomination.

Table 2: Factor of safety results achieved from PLAXIS analyses

Analysis Case	E_{ref} (MPa) (Drained)	E_{ref} (MPa) (Undrained)	FOS	
			Undrained	Drained
1	1.7	2	1.096	1.955
2	2.5	2.9	1.096	1.958
3	3.3	3.9	1.097	1.959
4	4.1	4.8	1.099	1.960
AVERAGE			**1.097**	**1.958**

The slope stability analyses were conducted using SLOPE/W under the Morgenstern-Price method and PLAXIS using the Phi-c reduction approach. Table 3 outlines the calculated FOS values from the proposed methods.

Table 3: Factor of safety results achieved from PLAXIS and SLOPE/W analyses

Code	Analysis Method	FOS	
		Undrained	Drained
SLOPE/W	Limit Equilibrium: Morgenstern-Price Approach	1.334	1.936
PLAXIS	Finite Element: Phi-c Reduction Approach	1.097	1.958
FOS Difference %		17.8%	1.1%

Undrained Analysis: The FOS result for undrained clay generated in SLOPE/W is higher than its PLAXIS equivalent. A number of researchers (Aryal, 2008; Ghiassian, 2009) also concluded that the inter-slice forces in a slope are more accurately calculated using PLAXIS, as the program takes the local stress distribution in the soil mass into account. Higher inter-slice shear forces produce a lower FOS. This is evident in the results of the undrained analysis. The LE method has limitations with regards to the computation of inter-slice shear forces. This is apparent where the stress concentrations are higher, i.e. where the slip circle fails. These differences give rise to higher safety factors predicted using the LE Morgenstern-Price approach than those obtained from the FE Phi-c reduction technique. Another reason contributing to the FOS difference between LE and FE results can be attributed to the geofabrics modelling procedure. In the LE method the mobilised geofabrics force is calculated as the minimum value between the pull-out resistance and the strength of the geofabrics. However in the FE analysis this force is calculated according to the strain level of the geofabrics.

Drained Analysis: In contrast to the undrained analysis, the drained analysis develops slightly larger FOS values in PLAXIS over its SLOPE/W counterpart. As can be seen in Table 3 the average FE method drained safety factor is only 1.1% higher than the Morgenstern-Price value. Therefore it can be concluded that the drained analysis FOS results are almost similar in each method and approximately identical critical slip surface shapes and locations were found.

It can be noted that both the limit equilibrium and FE methods have their own benefits and limitations. Both methods should be viewed as providing an estimation of the safety factor and critical slip surface. Although other studies have suggested that the FE method provides greater benefits than the traditional limit equilibrium method, the practicality and usability of the Morgenstern-Price approach is highly regarded among industry users. The use of the limit equilibrium method is much simpler, requiring less effort and therefore saves time in establishing a slope model. This benefit outweighs the use of the FE method as it requires an increased amount of time to input the necessary parameters and using the correct procedure to perform the calculations. In addition, geotechnical engineers should understand the limitations of each method and assess their results accordingly. It is recommended that users choose the program that best fits the slope stabilisation analysis that they intend to undertake. For example, when a simple slope is under assessment, the Limit Equilibrium technique is favoured. On the other hand, when the available input parameters are more advanced and the slope requires an acknowledgement of time (i.e. for construction or consolidation), the FE method is considered more suitable.

5 CONCLUSION

In this study attempt made to identify the limitations and advantages of finite element (FE) and limit equilibrium (LE) methods of slope stability analysis as guidance for practicing geotechnical engineers. A limit equilibrium based program, SLOPE/W, and FE based software, PLAXIS, were the respective software programs used to study the stability of various slopes. The findings indicated that when a simple homogeneous slope is considered, the difference in the safety factors and locations of the critical slip surface are minimal and both methods generate indistinguishable results. On the other hand, this study corroborates with previous studies that for differences between the FE and limit equilibrium methods are evident when a heterogeneous slope is analysed. With regards to the cases identified in this study for the undrained slope stability analysis, PLAXIS produced a FOS 17.8% lower than the corresponding SLOPE/W result. Although the locations of the critical slip surfaces were similar, there were differences in their shapes. This has been due to the use of different calculation methods (i.e. Phi-c reduction approach versus Morgenstern-Price method and ability of EF to distribute the loads during analysis). It is apparent that regarding an undrained analysis, the Morgenstern-Price approach is more conservative than the FE Phi-c reduction method. When drained conditions were considered, the FE method calculated the safety factors close to, but slightly larger than the limit equilibrium values. Both methods determined almost identical critical slip surface shapes and locations. Furthermore the effect of elastic modulus (E) was also taken into consideration. It is found that when the variation of the E is incorporated, it has an insignificant effect on the predicted FOS results, even though it does influence the computed deformation before failure. Thus, the slope geometry, the soil unit weight, the strength parameters (c and ϕ) and forces acting on the slope are the most important factors in both limit equilibrium and FE slope stability analyses.

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